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Modelling heat exchangers for thermoelectric generators

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Abstract

In order to further studies on thermoelectric generators, an analysis of the influence of fluid flow rate, heat exchanger geometry, fluid properties and inlet temperatures on the power supplied by the thermoelectric generator has been done. Different expressions and graphs showing this influence are shown in this paper, in order to give some practical guidelines for the thermoelectric generators design.

Keywords: Thermoelectric generators

1. Introduction

The thermoelectric power generator, as shown in Fig. 1, consists of a Peltier module sandwiched between two heat exchangers, hot and cold that create a temperature distribution through the module "t_1–t_2".

By virtue of this temperature difference ($\Delta t = t_1 - t_2$) the module supplies electrical power (W), this phenomenon is known as Peltier effect. Although, a lot of work has been done in this area, traditionally efforts have mainly been focused on the improvement of the thermoelectric properties of the materials and new techniques and guidelines for optimising the Peltier module geometry [1,2] in terms of a higher power output but always with Δt supposedly known. However, little attention has been paid on how the temperature difference Δt is obtained and what parameters affect it.

Thus, when a new thermoelectric generator design is planned, it is found that a tedious work to determine Δt has to be done. In order to facilitate this task, an easy and compact expression for Δt is given in this paper.

This study would remain incomplete, if the power consumed by the pump driving the fluid through the circuit and the thermoelectric generator was not considered. The pump consumption is proportional to the pressure drop through the circuit in which the pressure drop in the generator ΔP is included. So, the higher the pressure drop, the bigger the pump consumption. Since the bare power is defined as the total power supplied by the thermoelectric generator minus the pump consumption, the design parameters will be those that provide the most appropriate pair of values for Δt and ΔP (maximum and minimum, if possible, respectively).

This work is intended to give some guidelines for determining the optimum operating parameters of thermoelectric generator.

2. Analytical expression

Consider the basic thermoelectric generator, shown in the Fig. 1. The cold water (hot water) enters the cold exchanger (hot exchanger) at a temperature t_{ci} (t_{hi}) and exits at a temperature t_{co} (t_{ho}).

As a result of this temperature difference between the cold and hot sinks put in contact each other through the Peltier module, a heat flux appears. The heat transfer mechanism, here present is the so called *conduction*. Conduction is the heat transfer inside the bodies or from one body to another in physical contact with it. It takes place at the molecular level and involves the transfer of energy from the more energetic molecules (higher temperature regions) to those with a lower energy level (lower temperature regions). At macroscopic level the heat flux (q) is proportional to the temperature gradient reached inside the body and it is given by

$$q = -k\frac{\mathrm{d}T}{\mathrm{d}n} \tag{1}$$

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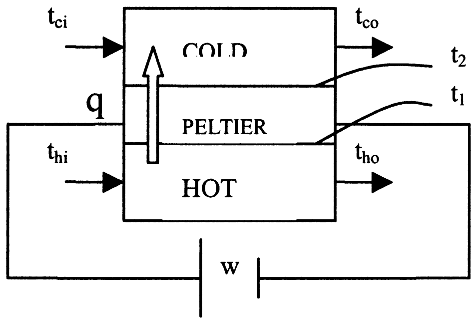

Fig. 1. Scheme of a thermoelectric generator.

Eq. (1) is the one-dimensional form of Fourier's law of heat conduction. Considering a one-dimensional heat flow, as it is the case in the Peltier module, along the x-direction in the plane wall shown in Fig. 2 direct application of Eq. (1) can be made, and then integration with the temperatures at both surfaces of the wall, t_1 and t_2, as boundary conditions yields

$$q = \frac{kA}{\Delta x}\Delta t \tag{2}$$

Applying Eq. (2) to the Peltier module we have got

$$q = \frac{k_p A_p}{e_p}\Delta t \tag{3}$$

where $\Delta t = t_1 - t_2$ is the temperature difference between the cold and hot plate of the module and k_p, A_p and e_p represent the thermal conductivity, surface area through which the heat is transferred and thickness of the Peltier module respectively. Eq. (2) can also be rewritten in the form

$$q = \frac{\Delta T}{\Delta x/kA} = \frac{1}{R}\Delta T \tag{4}$$

where $\Delta x/kA$ assumes the role of thermal resistance through the wall and is equivalent to the Ohm's law in electric circuits theory Fig. 2(b).

The W denotes the power output supplied by the generator and is function of Δt.

By using the electric analogy described above the heat transfer through the thermoelectric generator Fig. 3(a) can be represented by its equivalent electric circuit Fig. 3(b) and (c), and according to it the heat flow rate flowing through the generator is given by: $q = (1/\mathfrak{R})(T_{in} - T_{out})$ where R is the circuit's equivalent resistance and ($T_{in}$$-$$T_{out}$) is the mean logarithmic temperature difference between the cold and hot water.

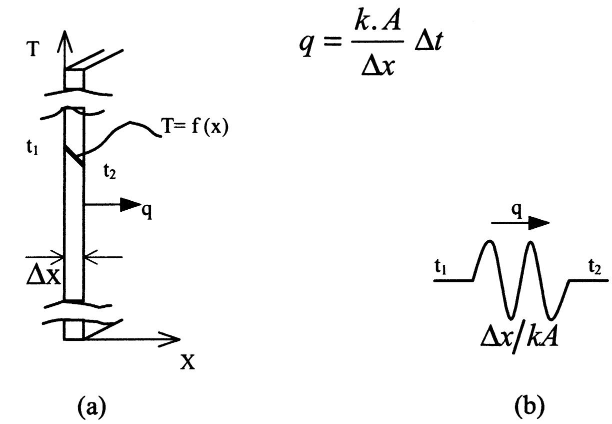

Fig. 2. One-dimensional heat conduction (a) and electric analogy (b).

Once the thermoelectric generator has been described, it is intended to develop an expression for the temperature difference Δt in which only the design parameters appear. These parameters include the exchanger geometry, flow rates, working fluid and inlet temperatures. In this paper, the working fluid will be water, hence this parameter is no longer a design parameter.

The last expression can also be written as [3]

$$q = \mathrm{UA}\,\Delta t_m \tag{5}$$

where $\mathrm{UA} = 1/\mathfrak{R}$ is the over-all heat transfer coefficient of the system and is defined as that factor for a given heat exchanger geometry, Peltier module and hydrodynamic conditions which, when multiplied by the mean temperature, yields the heat flux through the system and is given, according to Fig. 3(b) and (c), by the formula

$$\mathrm{UA} = \frac{1}{\mathfrak{R}} = \frac{1}{R_{h1} + R_p + 2R_{cop} + 2R_{paste} + R_{h2}} \tag{6}$$

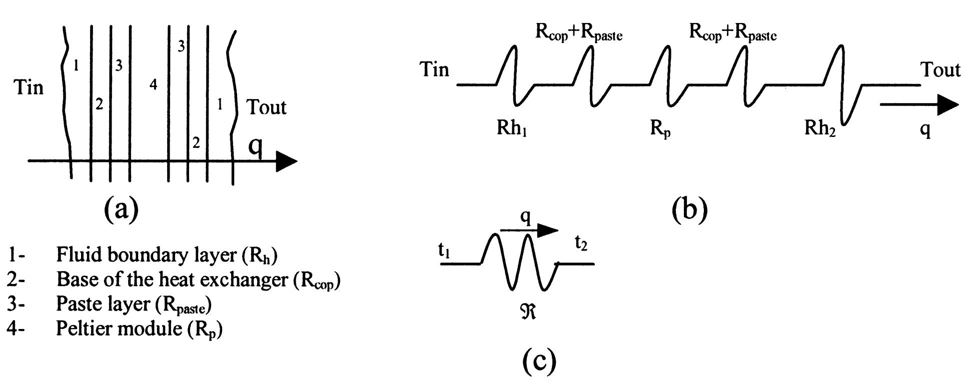

Fig. 3. Physical heat transmission (a); electric circuit (b) and (c).

where R_{h1} and R_{h2} are the thermal resistances by convection in the hot and cold exchangers ($R_h = 1/\eta hA$: being h the film coefficient of convection and η a parameter that depends on the heat exchanger surface area in contact with the fluid A_f) and R_p, R_{cop} and R_{paste} the thermal resistances by conduction of the module, heat exchanger and the paste between the module and the exchanger, respectively ($R = \Delta x/kA$). Δt_m has the form given by the Eq. (7), if a parallel-flow heat exchanger of simple pass is considered [3].

$$\Delta t_m = \frac{(t_{hi} - t_{ci}) - (t_{ho} - t_{co})}{\ln(t_{hi} - t_{ci})/(t_{ho} - t_{co})} \tag{7}$$

In a steady-state, the heat transferred from the hot to the cold fluid is the same as that passing through the Peltier module. Therefore, Eq. (3) must be equal to Eq. (5) from where it is easily determined the Δt

$$\Delta t = \mathrm{UA}\,\Delta t_m \left(\frac{e_p}{k_p A_p}\right) \tag{8}$$

However, the this last expression, so defined is not useful because it involves the exit fluid temperatures t_{ho} and t_{co} which are unknown before hand. Therefore, it will be necessary to redefine it by replacing t_{ho} and t_{co} for some known factors.

In order to do this, the three dimensionless factors have been used, namely, the number of transfer units NTU, effectiveness ε and capacity ratio C_R [3], which are defined as follow:

$$\mathrm{NTU} = \frac{\mathrm{UA}}{\dot{m}_h C_{ph}} \tag{9}$$

$$C_R = \frac{\dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} \tag{10}$$

$$\varepsilon = \frac{t_{hi} - t_{ho}}{t_{hi} - t_{ci}} \tag{11}$$

where $\dot{m}C_p$ denotes the amount of heat transferred to or from either fluid (cold or hot) per degree of temperature rise in the fluid ($\dot{m}$ is the mass flow rate and C_p the specific heat of the fluid). Effectiveness denotes the ratio of the real heat transfer rate from the hot to the cold water $\dot{m}C_p(t_{hi} - t_{ho})$ in the generator to the maximum possible heat transfer rate thermodynamically permitted $\dot{m}C_p(t_{hi} - t_{ho})$ and NTU is the ratio of the overall conductance UA to the smaller heat capacity rate $\dot{m}C_p$.

These three parameters do not involve any new quantity because the effectiveness can be determined as an explicit function of C_R and NTU for a given system. In this particular case (parallel-flow and single pass) [3]

$$\varepsilon = \frac{1 - e^{-\mathrm{NTU}(1+C_R)}}{1 + C_R} \tag{12}$$

Equalising this new definition of effectiveness Eq. (12) and the previous one Eq. (11), the hot outlet temperature may be directly obtained

$$t_{ho} = t_{hi} - (t_{hi} - t_{ci})\frac{1 - e^{-\mathrm{NTU}(1+C_R)}}{1 + C_R} \tag{13}$$

which determines the hot outlet temperature for any set of inlet temperatures when the exchanger geometry UA and the fluid flow rate and heat capacities $\dot{m}C_p$ are given.

When a steady state has been reached, the heat transferred from the hot fluid is approximately equal to that transferred to the cold fluid and the following equality can be written

$$\dot{m}_h C_{ph}(t_{hi} - t_{ho}) \approx \dot{m}_c C_{pc}(t_{co} - t_{ci}) \tag{14}$$

Now, taking into account the expression deduced for t_{ho} and Eq. (14), we obtain that

$$t_{co} = t_{ci} + C_R(t_{hi} - t_{ci})\frac{1 - e^{-\mathrm{NTU}(1+C_R)}}{1 + C_R} \tag{15}$$

Once the exit temperatures have been expressed as a function of some known parameters (design parameters) the Eq. (17) for the mean temperature difference Δt_m can be rewritten by introducing Eqs. (13) and (15) in it in the following form

$$\Delta t_m = (t_{hi} - t_{ci})\frac{1 - e^{-\mathrm{NTU}(1+C_R)}}{(1 + C_R)\mathrm{NTU}} \tag{16}$$

Therefore, introducing Eq. (16) into the Eq. (8) Δt takes the following form:

$$\Delta t = \dot{m}_h C_{ph}(t_{hi} - t_{ci})\left(\frac{e_p}{k_p A_p}\right)\frac{1 - e^{-\mathrm{NTU}(1+C_R)}}{1 + C_R} \tag{17}$$

The expression for pressure drop ΔP in pipes is [4]

$$\Delta P = f\frac{v^2}{2}\rho \tag{18}$$

where f, ρ and v are the loss coefficient, fluid density and fluid velocity through the heat exchanger, respectively.

3. Influence of the design parameters on Δt

3.1. Heat exchanger geometry

Any change in the heat exchanger geometry will be reflected in a change of the over-all heat exchanger coefficient UA in the sense that some of its thermal resistances will vary. The purpose of a geometrical change is to increase UA coefficient by varying the surface area of the heat exchanger in contact with the circulating fluid A_f.[1] This change of A_f also brings implicitly, a change in the film coefficient h which together with the new A_f results in a reduction of the thermal resistance by convection $R_h = 1/(hA_f)$ and as a result an increase of the UA.

[1] Note that the working fluid is water.

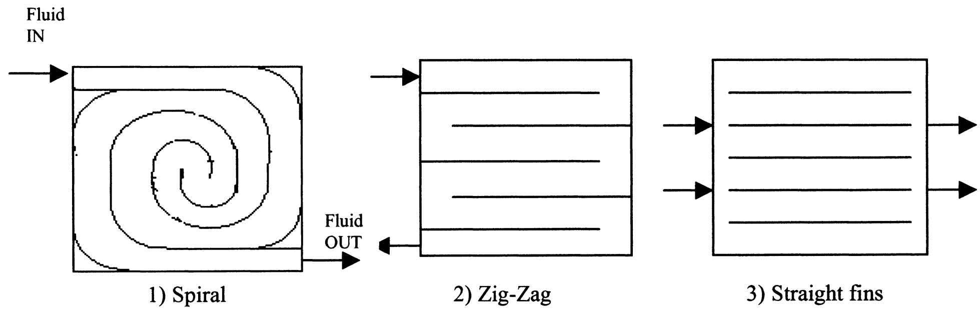

Fig. 4. Heat exchanger geometries.

In this section, three different geometries have been studied and are shown in Fig. 4

The values of UA obtained for each geometry are: spiral 2.07 W/C; zig-zag 2.094 W/C; straight fins 1.72 W/C.

Considering these three geometries and leaving constant the rest of the parameters, the result obtained for the temperature difference through the module is shown in Fig. 5.

Where ΔP is another parameter to be considered, when designing a thermoelectric generator because the bare power output supplied by the generator will be the total power output minus the power consumed by the pump driving the fluid through it.

As the pump consumption is a function of this ΔP, the geometry with lower ΔP should be chosen.

As it can be seen in Fig. 5, the maximum Δt is reached with the zig-zag geometry (2) but at the same time has the highest pressure drop ΔP.

Therefore, it is up to the thermoelectric generator designer to decide which pair of values Δt and ΔP best meet the operating conditions and as a consequence select the appropriate geometry.

3.2. Fluid flow rate and inlet temperatures

In this section, we study the influence of the flow rate and inlet temperatures on Δt for spiral heat exchanger geometry.

When varying the fluid flow rate, we indirectly modify the film coefficient and as a consequence the thermal resistances by convection is changed. Therefore, all this process results in a change in the value of the over-all heat transfer coefficient "UA" which, through the Eq. (17), yields a new value of "Δt".

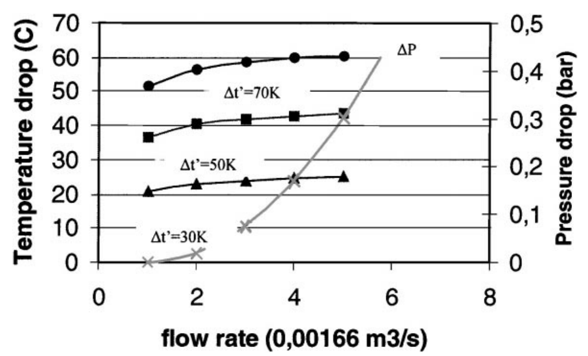

Fig. 6. Temperature and pressure drop through the module vs. flow rate.

The Fig. 6 shows this variation of "Δt" with respect to the fluid flow rate.

As can be seen from the Fig. 6, for a given inlet temperatures difference Δt, the temperature drop through the module increases with increasing the flow rate as well as the pressure drop. Also for a fixed flow rate the bigger the inlet temperature difference, the higher the Δt. Thus, according to the Fig. 6, it is more convenient to increase the inlet temperatures difference rather than the flow rate as it is obtained higher Δt and lower ΔP.

The Fig. 7 further highlights the influence of the inlet temperatures on the temperature difference through the module.

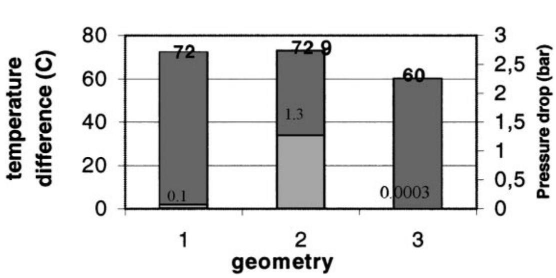

Fig. 5. Temperature difference and pressure drop vs. geometry.

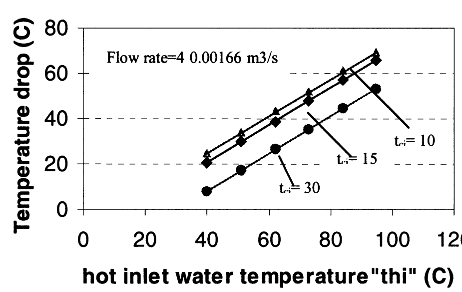

Fig. 7. Temperature drop vs. hot inlet temperature.

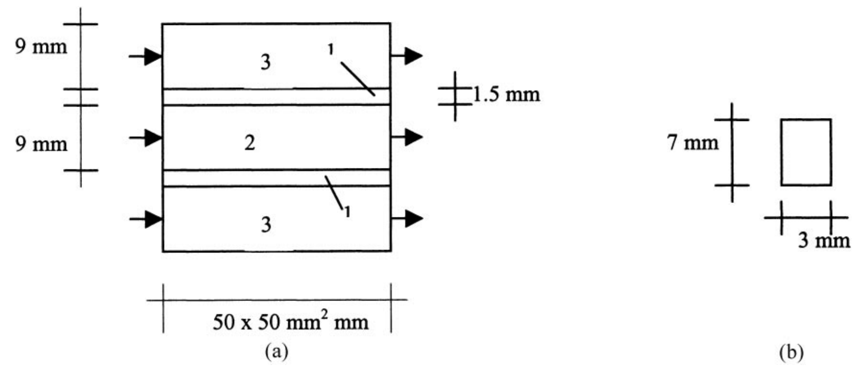

Fig. 8. Thermoelectric generator profile (a): (1) Peltier module; (2) hot heat exchanger; (3) cold heat exchanger and (b) the cross-sectional area of the spiral heat exchangers where the fluid flows through.

4. Experimental results

Until now, all results for the temperature through the module have been theoretically obtained by using the Eq. (17) and assuming some simplifications (water thermal properties constant with the temperature and no lateral heat flux through the spiral walls).

Therefore, it is necessary to check, whether the theoretical results meet the experimental values.

To do this comparison, the experimental tests were carried out with a thermoelectric generator as shown in Fig. 8.

The results obtained are shown in Fig. 9.

It is seen from the Fig. 9 that higher the inlet temperatures difference more the theoretical data deviate from the experimental data. As a whole the theoretical data compare quite well the experimental results, therefore, the simplifications made in this paper seem to be quite reasonable.

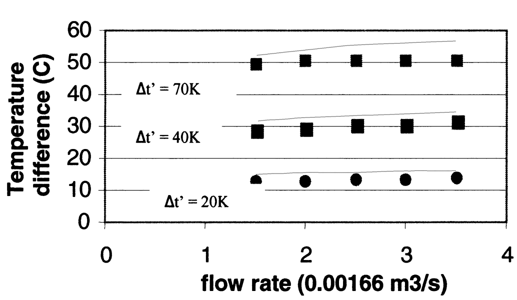

Fig. 9. Theoretical and experimental results of the temperature difference through the module vs. flow rate for different inlet temperature difference. The lines denotes the theoretical results and the points the experiment values.

It is also seen that as the flow rate increases the deviation between the theoretical and experimental data becomes larger. An explanation of it might be that the influence of the lateral heat flux, not considered here, is becoming considerably more important as the flow rate increases, or others factors may co-exist.

5. Conclusion

The expression developed here is an useful tool for those who have to design thermoelectric generators, since it will give them an idea about which operating conditions best meet the specifications required for a particular application.

The theoretical results meet well with the experimental values for low flow rates but not for high flow rates. This is because for high flow rates the parallel heat flux takes a major importance within the global heat transfer produced in the generator resulting in a decrease of the temperatures. It is true that the more accurate the over-all heat transfer coefficient "UA", the better the eventual results.

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